Dear future AP Physics students,

This an information and practice packet for a quiz that will be given sometime in the first two weeks of school. It does not need to be turned in.

Here’s the short story: Physics is NOT a math class. But you can’t do college-level physics without math. So I need you to be solid in the basic math techniques that we use frequently in AP Physics (most of which you’ve seen before). That’s what this packet is for.

Enjoy your summer!

Major Skills

To succeed in AP Physics, you will need to master the following skills. By "master", I mean you'll need to use them confidently without having to stop and think or refering to equations.

1. Scientific Notation.
2. Units: converting units, applying metric prefixes, and finding the units that result from mathematical operations.
3. Algebra: Solving single and simultaneous equations.
4. Geometry/Trig: Breaking angled quantities into right-angle quantities, and recombining right-angle quantities into angled results.
5. Basic physics: Finding density from basic amounts, and finding amounts from density.

Skill 1: Scientific Notation

It's assumed that you're already comfortable with scientific notation. If you aren't, then AP Physics probably isn't the right course for you. You will need to carry out many detailed calculations using scientific notation. You will need to develop the ability to quickly perform "order of magnitude" calculations using only the powers of ten. This will help you to anticipate the approximate size of the answer and ensure that your more detailed calculations are correct.

Skill 2: Units

<u>Manipulating Units:</u>

In physics, we have common units in which we measure variables. For example, displacement is measured in meters (m), time is measured in seconds (s), and mass is measured in kilograms (kg). Units just multiply and divide algebraically, so velocity (which is displacement divided by time) has the units of m/s.

<u>Metric Prefixes:</u>

To measure different quantities conveniently, we modify the basic units using metric prefixes. You can certainly apply the prefixes using Unit Conversions (see below), but it's preferable to be able to decode the prefixes without doing a formal conversion.

Factor	Prefix	Symbol
10^9	Giga	G
10^6	Mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

To convert a metric measurement quickly, first memorize the table of prefixes. Then use your head! You need a lot of small things to equal fewer big things. That will tell you whether to multiply or divide by the factor.

Example: to convert 4.7 km to meters, the factor is 10^3. Do we multiply or divide? Well, meters are smaller than km, so we need a lot of meters to make up a km. It makes sense to multiply by 10^3. We end up with 4700 m.

Example 2: to convert 80 nm to meters, the factor is 10^{-9}. Meters are bigger than nm, so I need a smaller number. I would multiply by 10^{-9}., making a smaller number (8×10^{-8} m).

1. How many centiliters (cL) in 150 liters?
2. How many kilometers (km) in 0.001 mm?
3. How many Newtons (N) in 12.68 x 10-5 GN?

<u>Unit Conversions</u>

Sample equalities: 1 meter = 39.37 inches
12 inches = 1 ft
1 pound = 4.448 newtons

The way that I advocate doing conversions is by setting up an algebraic equation. For instance, let's say that I want to convert 20 inches to nanometers.

$$20 inches\left(\frac{1 meter}{39.37 inches}\right)\left(\frac{1 nanometer}{10\text{^}-9 meters}\right) = 5.08 \times 10^8 \text{ nanometers}$$

Notice that the first number and the last number are physically the same quantity. The items in parentheses are mathematically equal to 1, since the numerators equal the denominators. You can see that the inches on top and inches on the bottom cancel out, and so do the meters, leaving you with nanometers

4. How many centimeters are in 48 inches?

5. How many inches are in 28nm?

6. How many pounds are in 600 Newtons?

Skill 3: Algebra

<u>Parametric analysis:</u>

In Physics, we have a variety of equations that show the relationship between several variables. We often need to figure out what changing one variable will do to another variable, without knowing specific quantities.

One common relationship is F = m a, or total force = mass * acceleration. Let's say that we push a block of mass m with a force of F and it accelerates with a value of a.

7. If we push another block with mass 2m, how much force is needed to have the same acceleration, a?

8. What if we now push a block of mass 3m with a force of 2F. What will the acceleration be? (in terms of a, like 2a, 0.5a, etc)

9. KE = ½ mv^2. How much would the kinetic energy of an object change if it has 3 times the initial velocity?

<u>Simultaneous Equations</u>

When we're solving problems involving systems of objects, we often write an equation for each object, ending up with multiple equations and multiple unknowns. To solve these equations, we need to combine them. There are a few ways to solve these problems. We'll only address the first 2 approaches here.

a. Rearrange one equation to solve one variable in terms of the other, then substitute into the other equation.
b. Multiply or divide one equation by a constant and then add or subtract equations to eliminate a variable.
c. Graph the equations, find the intersection.
d. Use matrices in the calculator.

Take a look at this example:
Solve for x and y:
5x -2y = 15
7x – 5y = 18

Substitution: The way that you were first taught to solve systems of equations was probably to do substitution; solve one variable in terms of the other and then substitute it in. This is most useful for simple equations.

If you solve for y in the first equation, you will get y = $\frac{5x}{2} - \frac{15}{2}$

Then substituting in y for the second equation, you find x = 3.54
Then by substituting x in for any of the above equation, you can find y =1.36
Check your answer by substituting your answers into the other equation; it should solve both equations.

Combining equations. Multiply each equation by a constant so one of the variables has a matching coefficient.

Starting with
5x -2y = 15

$7x - 5y = 18$

I'll multiply the top equation by 5 and bottom by 2 so the y variable has a coefficient of -10 in each equation.
$25x - 10y = 75$
$14x - 10y = 36$

Subtracting the bottom equation from the top, I get
$11x = 39$
So I can solve for x, and substitute that back into either equation to get y.

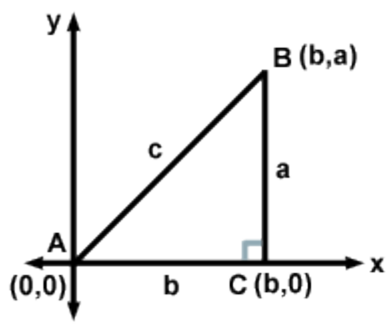
On a separate sheet of paper, solve these problems **and check your answer.** You can use whichever method you prefer.

10. $5x + y = 13$
    $3x = 15 - 3y$

11. $2x + 4y = 36$
    $10y - 5x = 0$

Skill 4: Geometry and Trigonometry

Consider the right triangle pictured below:



Using the lengths of the sides of right triangles such as the one above, the trigonometric functions can be defined in the following way:

$$a^2 + b^2 = c^2$$

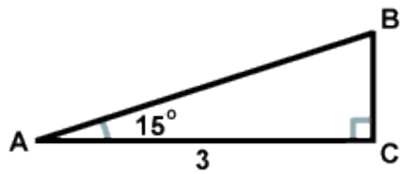
$$\sin(A) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos(A) = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

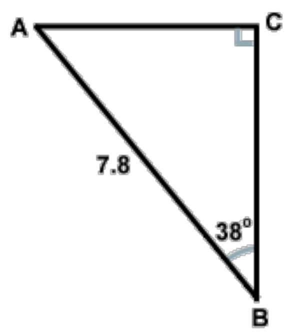
$$\tan(A) = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$

Find the other lengths of these triangles using the trig functions and/or the Pythagorean theorem. Show your work on a separate sheet of paper:

12.

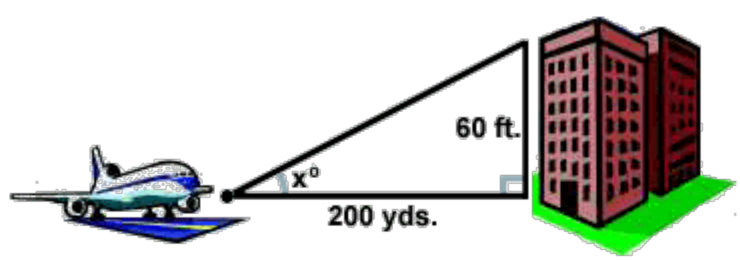


13.



If we know the sides, we can determine the angles. In the triangle above, if we know AC and AB, we know the sine of angle ABC is AC/AB (opposite over hypotenuse). So the angle ABC is $\sin^{-1}$ (AC/AB) .

14. An airplane takes off 200 yards in front of a 60 foot building. At what angle of elevation must the plane take off in order to avoid crashing into the building? Assume that the airplane flies in a straight line and the angle of elevation remains constant until the airplane flies over the building.

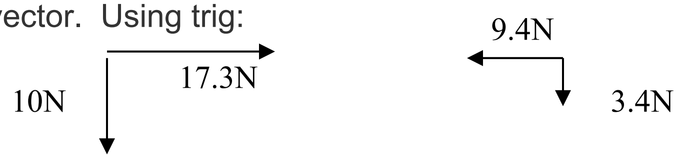


Vectors

In Physics, vectors are quantities that have direction. For example, temperature is not a vector because it doesn't have a direction. Forces are vectors, because it matters which direction the forces push. (Example: 32 N at an angle of 27 degrees)

One way we add vectors is by separately adding horizontal components (how much the vector goes horizontally) and vertical components. The vector sum is called the resultant.

To combine these two vectors, we would first find the components of each vector. Using trig:



So there would be a total of 7.9 N to the right and 13.4N down. Using the Pythagorean theorem, and finding the inverse tangent of 13.4/7.9, that would simplify to a 15.6 N force at an angle of about 60 degrees below horizontal.

15. A plane is flying 50 m/s in a direction 30 degrees north of east (in other words, northeast, but 30 degrees away from due east). A sudden wind starts blowing at 15 m/s in a direction 50 degrees south of east. Draw a picture of the situation, break the vectors into east/west components and north/south components, add the components, and combine them using the Pythagorean theorem. The result will be the final speed and direction of the plane.

16. Add the following vectors and determine the resultant.
    3.0 m/s, 45 deg and 5.0 m/s, 135 deg

17. Add the following vectors and determine the resultant.
    5.0 m/s, 45 deg and 2.0 m/s, 180 deg

Skill 5: Density

Population density is the number of people per unit area. In physics, density functions are often used for charge densities, mass densities and current densities. Density is a measure of stuff per unit space. The one you are most familiar with is mass density is mass/volume. You can also have one and two dimensional densities.

Linear mass density $\lambda = \frac{m}{l}$ (mass/length) λ = lambda

Surface mass density $\sigma = \frac{m}{A}$ (mass/ area) σ = sigma

Volume mass density $\rho = \frac{m}{V}$ (mass/volume) ρ = rho

You can also replace the mass (m) with charge (q) to determine charge densities or current (I) to determine current densities. So volume charge density is the charge per unit volume.

Volume of a sphere: $\frac{4}{3}\pi r^3$

Surface area of a sphere: $4\pi r^2$

Volume of a cylinder= $\pi r^2 h$

Surface area of a cylinder: $2\pi rh$ (for the sides) + $2\pi r^2$ (for the ends)

18. An iron sphere has a mass density of $\rho = 7.86 \times 10^3$ kg/m^3. If the sphere has a radius of 0.5 m, how much mass does the sphere contain?

19. A sphere made out of material x has a mass of 5 kg and has a radius of 4m. How much mass does a sphere of the same material with a 3 m radius have? (hint: since they are the same material, they have the same density)

20. A sphere made out of material y has a mass of 6 kg and has a radius of 3m. How much mass does a cylinder of the same material with a 4 m radius and a 2 m length have?

21. A cylinder (height 0.3m, radius 0.1m) has 2 Coulombs of charge spread out on its surface. If a sphere with the same charge had the same surface charge density, what would its radius be?

General Mathematical Reasoning Questions

22. Let's say the Earth's crust has an average thickness of 30 km. If the Earth's radius is $6x10^6$ m, what is the volume of the crust? (hint: the answer is not $1.36x10^{19}$ cubic meters, although that's close)

23. How many tennis balls would fill your bedroom?

24. Make an intelligent estimate for the number of high school teachers in the US. Show your work. (US population is about 300 million)

1. $150\,L \cdot \frac{1\,cL}{10^{-2}\,L} = 15000\,cL$

2. $0.001\,mm \cdot \frac{10^{-3}\,m}{1\,mm} \cdot \frac{1\,km}{10^{3}\,m} = 10^{-9}\,km$

3. $12.68\times10^{-5}\,GN \cdot \frac{10^{9}\,N}{1\,GN} = 12.68\times10^{4}\,N = 1.268\times10^{5}\,N$

4. $48\,in \cdot \frac{2.54\,cm}{1\,in} = 121.92\,cm$

5. $28\,nm \cdot \frac{10^{-9}\,m}{1\,nm} \cdot \frac{39.37\,in}{1\,m} = 1.1023\times10^{-6}\,in$

6. $600\,N \cdot \frac{1\,lb}{4.448\,N} = 134.89\,lb$

7. $F = ma$ If a doesn't change, but mass doubles, F will double

8. $a = \frac{F}{m}$ If you double the numerator and triple the denominator, $a' = \frac{2}{3}a$ where a' is the new accel.

9. $KE = \frac{1}{2}mv^2$ If $v' = 3v$ $KE' = \frac{1}{2}m(3v)^2 = \frac{1}{2}m9v^2 = 9\left(\frac{1}{2}mv^2\right) = 9KE$

10.
$$\begin{aligned} 5x + y &= 13 \\ 3x + 3y &= 15 \end{aligned} \quad \times 3$$

$$\begin{array}{rl} & 15x + 3y = 39 \\ - & (3x + 3y = 15) \\ \hline & 12x + 0 = 24 \end{array}$$

$x = 2,\ y = 3$

Or, in 1^{st} eq, $y = 13 - 5x$
Sub into 2^{ND} equation
Solve for X

11.
$$\begin{aligned} 2x + 4y &= 36 \quad \times 5 \\ -5x + 10y &= 0 \quad \times 2 \end{aligned}$$

$$\begin{array}{rl} & 10x + 20y = 180 \\ + & -10x + 20y = 0 \\ \hline & 0 + 40y = 180 \end{array}$$

$y = 4.5 \quad x = 9$

12. 3 = adjacent

$\cos 15 = \frac{3}{hyp}$ $Hyp = 3.106$ $Opp^2 = Hyp^2 - adj^2 = .804$

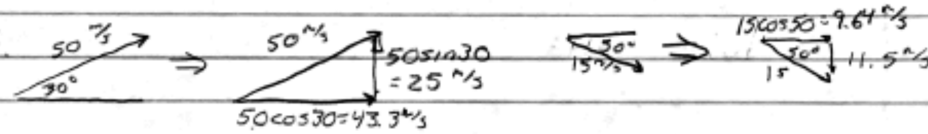
or $\tan 15 = \frac{opp}{3}$ $opp = 3\tan 15 = .804$

13. $\sin 38 = \frac{opp}{7.8}$ $opp = 4.8$

$\cos 38 = \frac{adj}{7.8}$ $adj = 6.15$

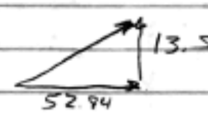
14. $\tan\theta = \frac{20\text{ yds}}{200\text{ yds}}$ $\theta = \tan^{-1}(.1) = 5.71°$

15.



Total E/W: 43.3 m/s + 9.64 m/s = 52.94 m/s E

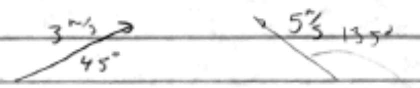
Total N/S: 25 m/s N + 11.5 m/s S = 13.5 m/s N



Total speed $= \sqrt{52.94^2 + 13.5^2} = 54.63$ m/s

Angle: $\tan^{-1}\left(\frac{13.5}{52.94}\right) = 14.3°$

16.



Horiz = 3cos45 − 5cos135 = −1.414 m/s

Ver = 3sin45 + 5sin135 = 5.66 m/s

Total speed: 5.83 m/s Angle: 104°

17. Total sp: 3.854 m/s Angle −66.5°

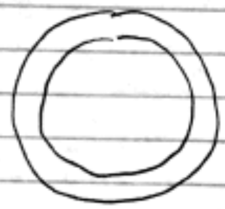
18. $\rho = \frac{m}{V}$ $\quad 7.86\times10^3 = \frac{m}{\frac{4}{3}\pi .5^3}$ $\quad m = 4115\ kg$

19. $\rho = \frac{m}{V} = \frac{5}{\frac{4}{3}\pi 4^3} = .01865 \frac{kg}{m^3}$

$.01865 = \frac{m}{\frac{4}{3}\pi 3^3}$ $\quad m = 2.109\ kg$

20. $\rho = \frac{6}{\frac{4}{3}\pi 3^3} = .0531 \frac{kg}{m^3}$ $\quad .0531 = \frac{m}{\pi 4^2 2}$ $\quad m = 5.\bar{3}\ kg$

21. $\sigma = \frac{q}{A} = \frac{2}{2\pi .1 .3 + 2\pi .1^2} = 7.958 \frac{C}{m^2}$ $\quad 7.958 = \frac{2}{4\pi r^2}$ $\quad r = .1414\ m$

22. 

$V_{earth} = V_{crust} + V_{INSIDE\ CRUST}$

$V_{crust} = V_{earth} - V_{inside\ crust}$

$= \frac{4}{3}\pi(6\times10^6)^3 - \frac{4}{3}\pi(6\times10^6 - 30000)^3$

$= 1.35E19\ m^3$

23. Hmm. If bedroom is about $4m \times 4m \times 3m = 48\ m^3$

And 12 balls fit in one linear meter, so 12^3 tb in $1\ m^3$

~ 83000 balls

24. Lets us Arlington. Pop ~ 200,000. About 1000 teachers

~ 1.5 million teachers

(Actually around 3 million. But thats OK - in making these kinds of estimates we want to at least get within one order of magnitude - that is one power of ten)